Example of Regex, NFA, DFA

Prof. James L. Frankel
Harvard University

Initial Regex

- Regex: ba*b(a|b)a

- Expressed in English
  - One "b" followed by zero or more "a"s followed by one "b" followed by either "a" or "b" followed by "a"

Construction of each NFA: expression a in ∑
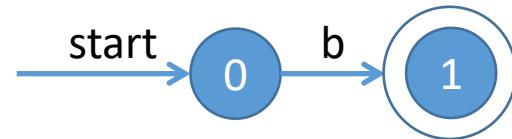
Regex: **b**a*b(a|b)a

Regex for: b

# Construction of each NFA: expression a in ∑

Regex: b**a***b(a|b)a
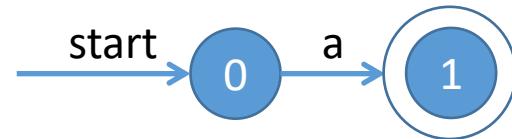
Regex for: a

Construction of each NFA: expression with closure (that is, *)

Regex: b**a***b(a|b)a
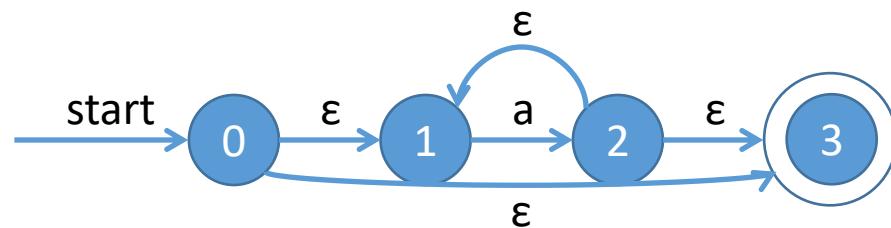
Regex for: a*

Construction of each NFA: expression a in ∑

Regex: ba***b**(a|b)a
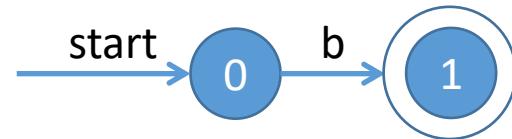
Regex for: b

Construction of each NFA: expression a in ∑

Regex: ba*b(**a**|b)a
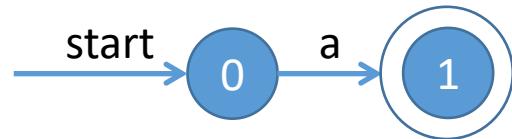
Regex for: a

Construction of each NFA: expression a in ∑

Regex: ba*b(a|**b**)a
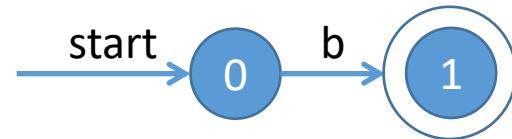
Regex for: b

# Construction of each NFA: expression with union (that is, |)

Regex: ba*b**(a|b)**a

Regex for: (a|b)
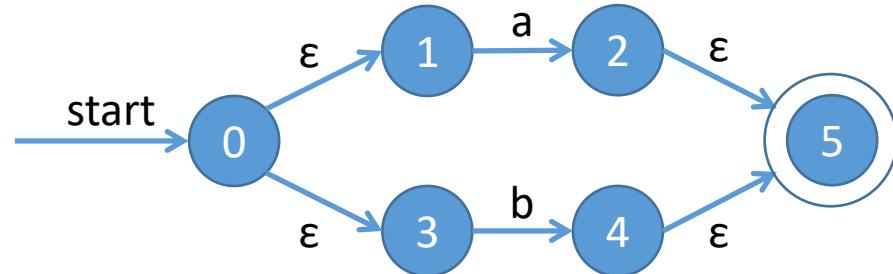
Construction of each NFA: expression a in ∑

Regex: ba*b(a|b)**a**
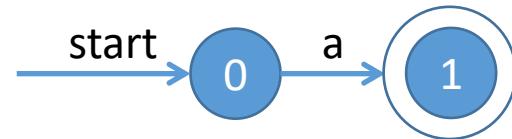
Regex for: a

# Construction of each NFA: expression with concatenation

Regex: **ba***b(a|b)a
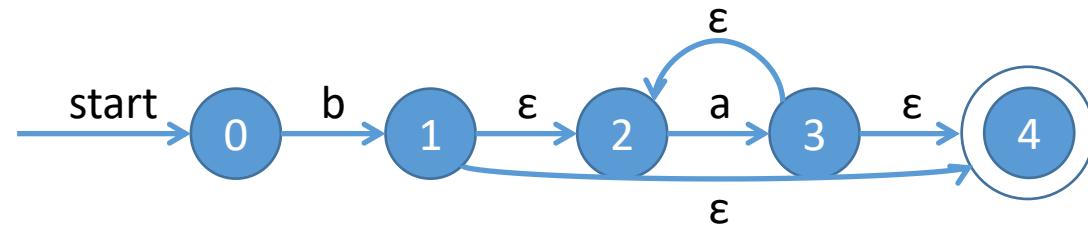
Regex for: ba*

Construction of each NFA: expression with concatenation

Regex: **ba*b**(a|b)a
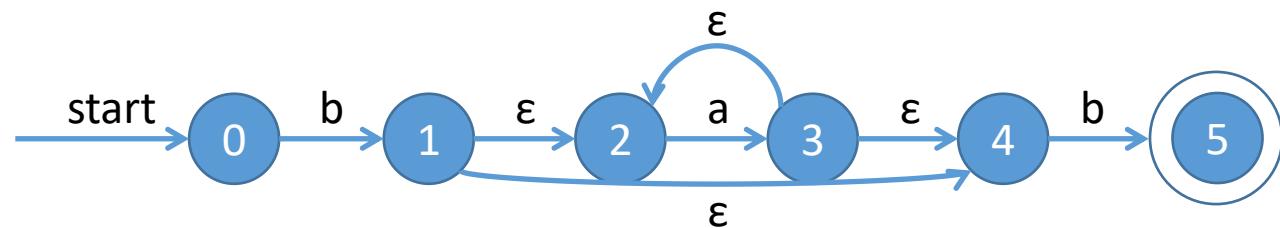
Regex for: ba*b

Construction of each NFA: expression with concatenation

Regex: **ba*b(a|b)**a
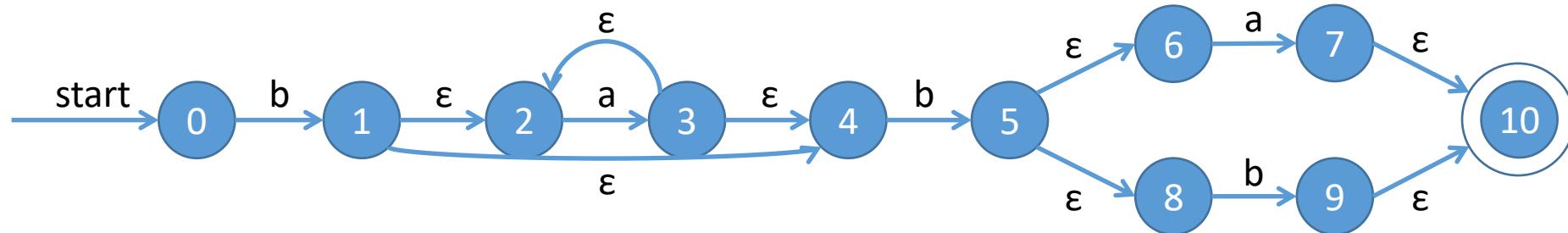
Regex for: ba*b(a|b)

Construction of each NFA: expression with concatenation

Regex: **ba*b(a|b)a**

Regex for: ba*b(a|b)a
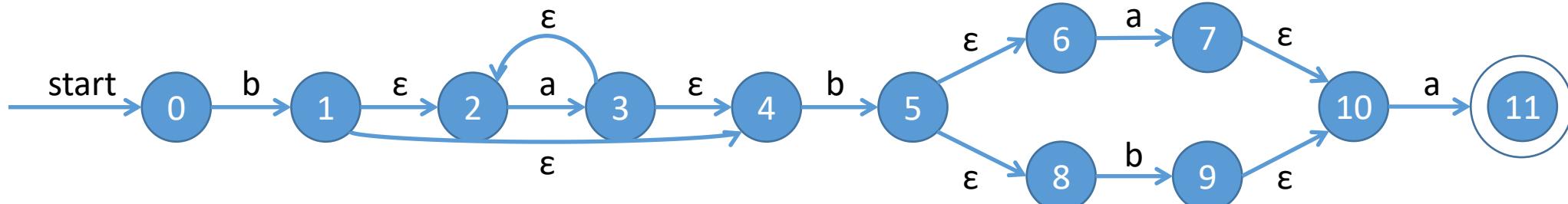
Final Complete NFA Constructed from Regex

Regex: ba*b(a|b)a
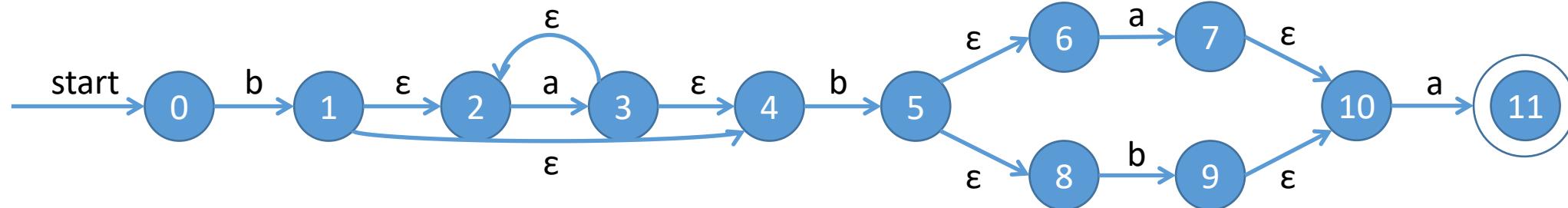
Subset Construction of DFA from NFA

Regex: ba*b(a|b)a

Unmarked state is indicated by red

Dstates = { ε-closure(s_0) } = { {0} }

Dstates = { {0}, ε-closure(move({0}, b)) } =

      { {0}, ε-closure({1}) } =

      { {0}, {1, 2, 4} }

Dtran[{0}, b] = {1, 2, 4}

Subset Construction of DFA from NFA

Regex: ba*b(a|b)a

Dstates = { {0}, {1, 2, 4}, ε-closure(move({1, 2, 4}, a)),

ε-closure(move({1, 2, 4}, b))} =

{ {0}, {1, 2, 4}, ε-closure({3}), ε-closure({5}) } =

{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8} }

Dtran[{1, 2, 4}, a] = {2, 3, 4}

Dtran[{1, 2, 4}, b] = {5, 6, 8}

Subset Construction of DFA from NFA

Regex: ba*b(a|b)a

Dstates = { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8},
$\quad$ ε-closure(move({2, 3, 4}, a)), ε-closure(move({2, 3, 4}, b)),
$\quad$ ε-closure(move({5, 6, 8}, a)), ε-closure(move({5, 6, 8}, b)) } =

$\quad$ { {0}, {1, 2, 4} , {2, 3, 4}, {5, 6, 8},
$\quad$ ε-closure({3}), ε-closure({5}),
$\quad$ ε-closure(move({5, 6, 8}, a)), ε-closure(move({5, 6, 8}, b)) } =

$\quad$ { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8},
$\quad$ ε-closure(move({5, 6, 8}, a)), ε-closure(move({5, 6, 8}, b)) }

Dtran[{2, 3, 4}, a] = {2, 3, 4}

Dtran[{2, 3, 4}, b] = {5, 6, 8}

Subset Construction of DFA from NFA

Regex: ba*b(a|b)a

Dstates = { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8},
 ε-closure(move({5, 6, 8}, a)),
 ε-closure(move({5, 6, 8}, b)) } =

 { {0}, {1, 2, 4} , {2, 3, 4}, {5, 6, 8},
 ε-closure({7}), ε-closure({9}) } =

 { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10} }

Dtran[{5, 6, 8}, a] = {7, 10}

Dtran[{5, 6, 8}, b] = {9, 10}

Subset Construction of DFA from NFA

Regex: ba*b(a|b)a

Dstates = { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10},
ε-closure(move({7, 10}, a)), ε-closure(move({7, 10}, b)),
ε-closure(move({9, 10}, a)), ε-closure(move({9, 10}, b)) } =

{ {0}, {1, 2, 4} , {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10},
ε-closure({11}),
ε-closure(move({9, 10}, a)), ε-closure(move({9, 10}, b)) } =

{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11} ,
ε-closure(move({9, 10}, a)), ε-closure(move({9, 10}, b)) }

Dtran[{7, 10}, a] = {11}

Subset Construction of DFA from NFA

Regex: ba*b(a|b)a

Dstates = { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11},
 ε-closure(move({9, 10}, a)), ε-closure(move({9, 10}, b)) } =

{ {0}, {1, 2, 4} , {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11},
 ε-closure(move({9, 10}, b)) } =

{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11} }

Dtran[{9, 10}, a] = {11}

Subset Construction of DFA from NFA

Regex: ba*b(a|b)a

Dstates = { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11} } =

{ {0}, {1, 2, 4} , {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11},

ε-closure(move({11}, a)), ε-closure(move({11}, b)) } =

{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11} }

Subset Construction of DFA from NFA: Resulting Dstates and Dtran

Dstates = { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11} }

Dtran =

D State	Input Symbol	New D State
{0}	b	{1, 2, 4}
{1, 2, 4}	a	{2, 3, 4}
{1, 2, 4}	b	{5, 6, 8}
{2, 3, 4}	a	{2, 3, 4}
{2, 3, 4}	b	{5, 6, 8}
{5, 6, 8}	a	{7, 10}
{5, 6, 8}	b	{9, 10}
{7, 10}	a	{11}
{9, 10}	a	{11}

Subset Construction of DFA from NFA: Renamed Dstates and Dtran

Dstates = { A, B, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11} }

Dtran =

D State	Input Symbol	New D State
A	b	B
B	a	C
B	b	D
C	a	C
C	b	D
D	a	E
D	b	F
E	a	G
F	a	G

Resultant DFA

- Expressed in English
  - One "b" followed by zero or more "a"s followed by one "b" followed by either "a" or "b" followed by "a"